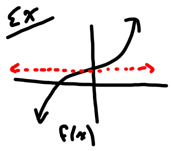
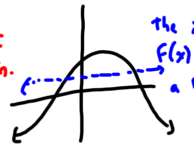
3.4 Inverses (f^{-1})

HLT (Horizontal Line Test) - Determines if the inverse of a relation will be a fn.

Ex



the inverse of f(x) is a fn.

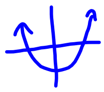


The inverse of f(x) is NOT a fn.

Ex $f(x) = x^2 - 4$

(a) Is the inverse of f(x) a fn? NO

Fails HLT



(b) $f^{-1}(x)$ $f(x) = y$

We can find $f^{-1}(x)$ by interchanging x & y

$f(x) = x^2 - 4$

$y = x^2 - 4$

$x = y^2 - 4$

$+4 \quad +4$

$\sqrt{x+4} = \sqrt{y^2}$

$y = \pm\sqrt{x+4}$

$f^{-1}(x) = \pm\sqrt{x+4}$

(c) Graph $f(x)$ & $f^{-1}(x)$

Ex $f(x) = |4x|$

All we do is interchange the x & y coordinates of the ordered pair of the original fn.

f(x)

x	y
-2	8
-1	4
0	0
1	4
2	8

$\leftrightarrow$

$f^{-1}(x)$

x	y
8	-2
4	-1
0	0
4	1
8	2

2 fn's, f & f^{-1}, are inverse fn's iff $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x)$

Ex $f(x) = 3x^2 + 7$

$f^{-1}(x) = \sqrt{\frac{x-7}{3}}$

$(f \circ f^{-1})(x)$

$f\left(\sqrt{\frac{x-7}{3}}\right) = 3\left(\sqrt{\frac{x-7}{3}}\right)^2 + 7$

$= \cancel{3}\left(\frac{x-7}{\cancel{3}}\right) + 7$

$= \boxed{x}$

$(f^{-1} \circ f)(x) =$

$f^{-1}(3x^2+7) = \sqrt{\frac{(3x^2+7)-7}{3}}$

$= \sqrt{x^2}$

$= \boxed{x}$

$\therefore$ f & f^{-1} are inverse fn's